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STARTING TIMES FOR DATA COLLECTION

IN A QUEUEING SIMULATION I: EXPERIMENTS

WITH A SINGLE SERVER MODEL

George S. Fishman and Louis R. Moore

Technical Report 78-1

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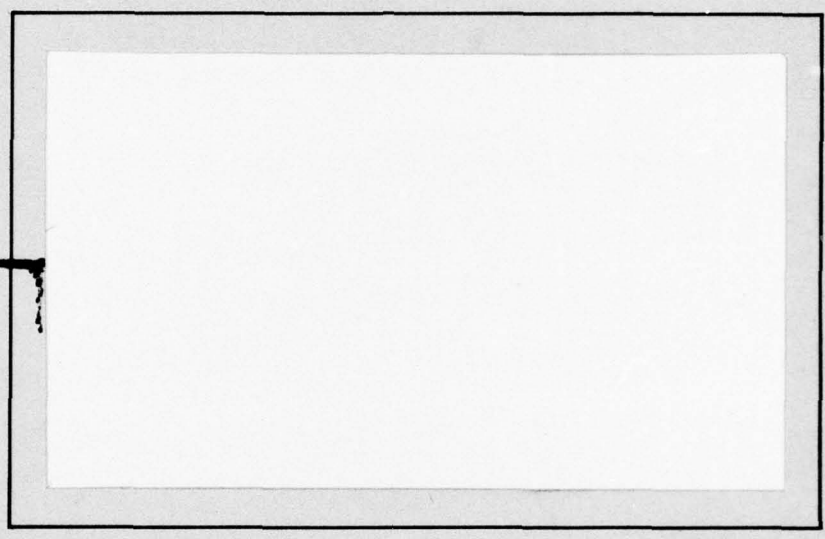
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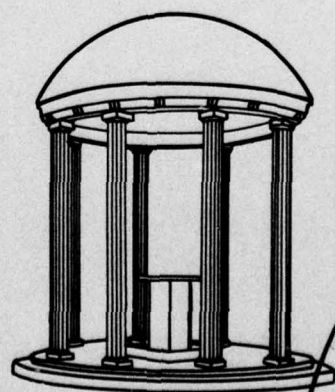
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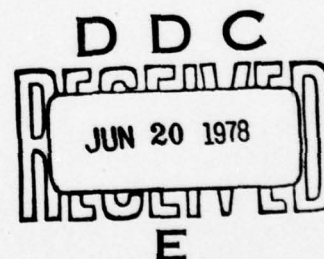
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ABSTRACT

This paper presents results of experimentation aimed at identifying suitable starting rules for discrete event simulations. A starting rule is a decision rule that tells a simulation analyst when to begin collecting data that are relatively free of the initial conditions of a simulation. The starting rules described here rely for decision making on a comparison between a priori information on interarrival and service times and corresponding sample quantities computed during the course of a simulation. Testing of the first proposed rule on a single-server queueing simulation with exponential interarrival and service times revealed a serious inadequacy. However, an examination of just how this inadequacy arose led to a second proposal for a starting rule. When tested in a parallel simulation the second rule produced considerably more favorable results. In addition, a perusal of the distribution of starting time for 1000 replications suggests a direction for future research aimed at reducing this starting time.

1. <u>Introduction</u>

Among statistical problems that arise in the course of running a discrete
event simulation, the effect of <u>initial conditions</u> on simulation output has long
occupied a prominent position. Initial conditions refer to the <u>state</u> of critical
variables at the beginning of a simulation run. Because of the dependence among
phenomena in a simulation and the temporal nature of much of this dependence,
the choice of initial conditions influences the time paths of these phenomena.
Since interest traditionally has focused on steady state behavior, research on
the topic of initial conditions has concentrated on ways of diluting their
influence on the sample records used for analysis. Gafarian, Ancker and
Morisaku (1977) have reviewed and evaluated the published proposals for
solving this problem. Their findings reveal that a viable solution remains
to be uncovered. The present paper provides a framework for research into this
topic and, based on initial findings, recommends procedures to overcome the
problem of initial conditions. The procedures rely on ancillary information
that is available to the simulation user and, when properly used, can induce
acceptable behavior in sample paths.

The analysis of queueing systems represents the most common use of discrete
event simulation. To begin a simulation run of a queueing system one needs to
specify the state of congestion that prevails at that moment. Since the purpose
of inquiry is to uncover congestion characteristics, one can hardly be expected
to meet this need with anything but frustration. Nevertheless, there is one
set of initial conditions whose effect on time paths is relatively straightforward.
If a simulation begins with an arrival to a system with no busy servers and no
queues one anticipates that the system will appear uncongested near the beginning

of the run. That is, queue lengths and waiting times appear shorter than one would expect in the steady state. Although this set of initial conditions leads to undesirable behavior, its ease of implementation relative to other initial conditions gives it greater appeal in practice. The procedures to be described here all begin with this initial condition which, as we show, can be made to pose no special problem.

To overcome the inadequacy of past proposals this paper presents an approach to solving the initial conditions problem that uses ancillary information available to an analyst before and during execution of a simulation of a queueing system. This information consists of arrival and service rates or their corresponding reciprocals, mean interarrival and mean service times. The principal idea is to compute sample quantities such as sample mean interarrival and service times and compare them with their corresponding true values. When the deviations between sample and corresponding true quantities fall within prespecified tolerances one begins data collection for purposes of statistical inference about the steady state. Prior to beginning the experimental part of this study, the rationale here was that making sample quantities representative of their corresponding time quantities would make the state of the system at the time that data collection commences representative of the true steady state. As our results will show, putting the system exactly in the steady state exceeds the capacity of our suggested rule. However, the rule does enable one to put a system into a state of above average congestion. This alternative capacity has value since if the choice lies between beginning data collection in an undercongested or overcongested system, most analysts would prefer the overcongested situation. This is especially true if the purpose of the analysis is to infer congestion characteristics.

Although one easily appreciates the notion of a data collection starting

rule based on a comparison of sample and theoretical values, several issues

need resolution before one can implement such a rule. The issues include:

1. Which quantities should enter the decision to start data
    collection?
2. What should the tolerance be for the comparison of sample
    and theoretical quantities?
3. What explicit form should the starting rule assume?
4. For which quantity does one wish to collect data?
5. How does one evaluate a rule's performance?
6. How does the tolerance affect the performance of the rule?

Although the present paper addresses each of these issues, one cannot claim

answers that apply across all conceivable queueing simulations. Nevertheless,

our findings do reveal procedures that should prove useful for many single

server queueing systems and conceivably for some multiserver system as well.

Section 2 presents a discussion of the aforementioned issues and formulates

rule 1, experimental results for which appear in Section 3 with evaluation. The

evaluation leads to the formulation of an alternative, rule 2, empirical results

for which also appear in Section 3 with a demonstration of its superiority over

rule 1. As the reader already may have noted, the aforementioned rules allow

for a variable number of observations to be omitted from a sample record at

the beginning of a simulation run. Because the experimental results indicate

that this number may be large, Section 4 formulates rule 3, a modification of

rule 2, that allows for a dramatic reduction in the number of unused observations.

The efficacy of this rule remains to be tested.

2. Problem Formulation

Consider a simulation model of a queueing system with m servers in

parallel, independent interarrival times with mean $1/\lambda$ and independent service

times with mean $1/\omega$. Let T_i denote the elapsed time between arrivals of

jobs i-1 and i and let S_i denote the service time of arrival i . Assume that the simulation begins with the arrival of job 1 to an empty queue and m idle servers. Let X_i denote the system time of completion i where system time denotes waiting time plus service time. Assume that an ultimate objective of analysis is to infer the characteristics of the <u>system time</u> stochastic process from a sample record of system times. Moreover, one wishes to conduct this analysis free of the undercongesting influence of the empty and idle initial conditions. Suppose the sample record is $X_{N+1}, X_{N+2}, \ldots, X_{N+M}$ where M is specified. Then the problem at hand is to determine N in a simulation run so that $X_{N+1}, \ldots, X_{N+M}$ each come from the steady-state probability distribution.

Selecting a Rule

After n completions occur during a simulation run one can estimate $1/\lambda$ and $1/\omega$ by $n^{-1} \sum_{i=1}^{n} T_i$ and $n^{-1} \sum_{i=1}^{n} S_i$ respectively. These estimates are unbiased and independent of initial conditions. In principle the closer the sample quantities are to their respective means, the more one inclines to believe that subsequent $X_{n+1}, X_{n+2}, \ldots$ each represent steady-state system times. Then one conceivable starting rule is

$$N = \min(n: \ |\sum_{i=1}^{n} T_i - n/\lambda| \leq \delta_1, \ |\sum_{i=1}^{n} S_i - n/\omega| \leq \delta_2) \qquad 0 < \delta_1, \delta_2 \ ,$$

where one needs to specify the tolerances δ_1 and δ_2 explicitly. We call N the <u>starting time</u> since data collection starts with completion N+1 .

At the outset of inquiry into starting rules, working with two tolerances would be difficult. An alternative system quantity of interest is the

activity level or traffic intensity

(1) ρ = arrival rate/no. of servers x service rate

                          = $\lambda/m\,\omega$,

for which one has an estimate

$$\tilde{\rho}_N = \sum_{i=1}^{N} S_i/m \sum_{i=1}^{N} T_i .$$

One starting rule based on (1) is

(2) $N = \min(n: |\tilde{\rho}_n - \rho| \leq \delta)$ $0 < \delta$.

The first starting rule that we test is closely related to (2).

For fixed   n   a considerable amount is known about $\tilde{\rho}_n$. We consider

one special, but important, case. Suppose that interarrival times have a p

stage Erlang distribution with mean $1/\lambda$ and that service times have a q stage

Erlang distribution with mean $1/\omega$. Then $q\tilde{\rho}_n/p\rho$ has an F distribution

with 2np and 2nq degrees of freedom.   Moreover,

$$E(\tilde{\rho}_n) = \rho np/(nq - 1)$$

This suggests that when working with Erlang distributions one use

$\hat{\rho}_N = \tilde{\rho}_N(nq - 1)/np$ for test purposes and consider

Rule 1 $N = \min(n: |\hat{\rho}_n - \rho| \leq \delta)$ $0 < \delta$.

We note in passing that rule 1 will not be the ultimate recommendation

that we make to potential users. However, a careful analysis of the results

for this rule is a prerequisite for guidance in identifying a better rule. This we do in Section 3.

Selecting a Tolerance Level δ

Let us now turn to the choice of δ . It is not difficult to anticipate an inverse relationship between N and δ . Also one expects that a smaller δ should make the steady state more of a reality for subsequent system times. Although we need to rely on experimentation to determine how δ affects this closeness to steady state one additional observation deserves attention. If one were to choose $\delta \equiv \alpha\rho$ for $0 < \alpha$ then N would have a distribution independent of ρ . This appears contrary to intention for one intuitively expects initial conditions to influence X_{N+1} less for, say, $\rho = .7$ than for $\rho = .9$. Therefore we avoid this approach to setting δ levels. Section 3 studies starting rules for $\delta = .0025, .005, .01, .02, .05$ and .1 .

Criteria for Evaluating a Starting Rule

One now needs a method for evaluating rule 1. In particular, one wants the rule to produce a sequence $X_{N+1}, X_{N+2}, \ldots$ of steady-state system times. For single server queueing models Lindley's equation tells us that [4]

$$(3) \qquad X_{N+1} = \max(0, X_N - T_{N+1}) + S_{N+1} ,$$

which implies that X_{N+1} is a function of X_N , T_{N+1} , and S_{N+1} . However, only X_N conceivably could depend on the initial conditions at the beginning of the simulation. Therefore, if X_N does not have the steady-state distribution one is skeptical that X_{N+1} does; if X_{N+1} does not have the steady-state distribution one is skeptical that X_{N+2} does, etc. Here one can regard X_N as the initial condition that determines the extent to which $X_{N+1}, X_{N+2}, \ldots$ have steady-state distributions and X_{N+1} as the observation most influenced by

this initial condition. The remainder of this paper concentrates on the distribution of X_{N+1} as the criterion of evaluation.

Tests Used for Evaluation

To test rule 1 we use a single server $(m = 1)$ model with Poisson arrivals and exponential service times. For specified λ, ω and δ we compare the empirical cumulative distribution function of X_{N+1} for 1000 independent relications with the theoretically known steady-state cumulative distribution function (c.d.f.) , which is exponential with mean $\rho/(1 - \rho)$ [4] .

Let N_j denote the starting time on replication j and X_{j, N_j+1} the system time of completion N_j+1 on replication j . Consider J replications with sample data $X_{1,N_1+1}, \ldots, X_{J,N_J+1}$ which are independent and identically distributed when using rule 1. We wish to test the hypothesis H_0: $X_{1,N_1+1}, \ldots, X_{J,N_J+1}$ have the c.d.f.

(4) $$F(x) = 1 - e^{-(1/\rho - 1)x} \qquad\qquad 0 \le x < \infty .$$

Under H_0 the statistics

$$Y_j = 1 - e^{-(1/\rho - 1)X_{j, N_j+1}} \qquad\qquad j = 1,\ldots,J$$

are independent uniform deviates with c.d.f.

$$G(y) = y \qquad\qquad 0 < y \le 1 .$$

The empirical c.d.f of the Y_j's is for all y

$$G_J(y) = \frac{1}{J} \sum_{j=1}^{J} I_{(0, y]}(Y_j) ,$$

I being the indicator function. To check H_0 one examines the deviations

$$\Delta(y) = G_J(y) - y \qquad\qquad\qquad 0 < y \leq 1 , \qquad .$$

or functions of these deviations. For the Kolmogorov-Smirnov goodness-of-fit test one uses the test statistic

$$D^* = \sup_y |\Delta(y)| \quad .$$

Critical values of the test statistic appear in Miller (1956) and Owen (1962). For the chi-square goodness-of-fit test with K equiprobable cells the statistic is

$$x^2 = JK \sum_{i=1}^{K} [\Delta(i/K) - \Delta(i/K - 1/K)]^2$$

which for large J has the chi-square distribution with $K - 1$ degrees of freedom under H_0 . For the Anderson-Darling test [1] one uses the statistic

$$W^2 = n \int_0^1 \{[\Delta(y)]^2/y(1 - y)\} dy \quad ,$$

which is particularly sensitive to departures of $G_J(y)$ from $G(y)$ in the tails of the distribution. Critical values of the test statistic appear in Lewis [5].

The aforementioned statistics test for departures from H_0 . In many cases, including the present, it is of interest to study the nature of the departure if H_0 is rejected. For us the most desirable alternative is to show that X_{1,N_1+1}, $\ldots,X_{J,N_J+1}$ have a distribution that is <u>stochastically</u> <u>greater</u> than the steady-state time distribution. Let W and V be random variables with c.d.f.'s F_W and F_V .

One says that W is stochastically greater than V if $F_W^{-1}(u) - F_V^{-1}(u)$

is non-negative and not identically zero on the open interval $(0,1)$, where

F_W^{-1} and F_V^{-1} denote the right continuous inverses of F_W and F_V respectively.

In the present case this means that the system times $X_{1,N_1+1}, \ldots, X_{J,N_J+1}$

exhibit more congestion than steady-state system times would show.

The Kolmogorov-Smirnov test allows one to check the hypothesis

$H_1: X_{1,N_1+1}, \ldots, X_{J,N_J+1}$ have a distribution that is stochastically

greater than the F in (4). The test statistic is

$$D^- = -\inf_y \Delta(y) \quad .$$

Dwass (1962) describes an additional helpful measure of discrimination. The
statistic

$$U = \int_0^1 I_{[-\infty, \, 0)}(\Delta(y))dy$$

gives the proportion of $G(y)$ that lies below $G(y) = y$. Under H_0 U has

the uniform distribution on $(0, 1)$. If H_1 is true one expects U to be

close to unity.

3. Experimental Results

This section begins with a presentation of results for rule 1 for system

time in a $M/M/1$ queueing simulation with

$$\omega = 1$$
$$\lambda = .7 \text{ and } .9$$
$$\delta = .0025, .005, .01, .02, .05, .1$$
$$J = 1000 \quad .$$

Each of the $1 \times 2 \times 6 = 12$ design points was replicated $J = 1000$ times and

on each replication of each design point N_j, X_{j,N_j+1} and $\hat{\rho}_{j,N_j}$ were recorded. To check on the correctness of the simulation 1000 replications with $N_j = 1000$ for $j = 1,\ldots,1000$ were run for each design point. On each replication system time was recorded.

Table 1 gives the goodness-of-fit statistics for the empirical c.d.f.'s of X_{j,N_j+1} for each of the design points. For tests of size $\alpha = .05$ the critical values of the Kolmogorov-Smirnov, Anderson-Darling and chi-square tests are .0430, 2.492 and 30.1 respectively. The results are dismal, H_0 being rejected in all cases but the fixed starting time case of $N = 1000$. This last observation merely confirms the fact that initial conditions hardly affect completion 1001 for $\rho = .7$ and $.9$. To obtain insight into the nature of the deviations, plots of $\Delta(y)/\sqrt{y(1-y)}$ were prepared for each c.d.f. in Figures 1 and 2 in the Appendix. Notice that for $\rho = .7$ the curves of the deviations become negative as δ decreases. By contrast for $\rho = .9$ the curves become more positive as δ decreases.

Although initially puzzling the cause of poor performance becomes apparent in Figure 3. It reveals that small system times usually occur with large $\hat{\rho}_N$'s (small $\rho/(\rho + \hat{\rho}_N)$) and large system times occur with small $\hat{\rho}_N$'s (large $\rho/(\rho + \hat{\rho}_N))^{\dagger}$. The explanation of behavior is now apparent. When a large $\hat{\rho}_N$ satisfies rule 1 it usually does so by entering the acceptance interval $[\rho - \delta, \ \rho + \delta]$ from above. This implies that completion N has either a short service time or a long interarrival time. From (3) it is clear that either of these manifestations inclines to reduce X_{N+1} through X_N. Conversely, a small $\hat{\rho}_N$ that satisfies rule 1 usually enters the acceptance interval from

$^{\dagger}F(X_{N+1})$, where (4) defines F, maps system time onto $(0, 1)$, as does the transformation $\rho/(\rho + \hat{\rho}_N)$.

Table 1
Goodness-of-Fit Statistics
for the Distribution of X_{N+1}

ρ	δ	Rule 1					Rule 1a					Rule 1b					Rule 2				
		D^*	D^-	X^2	U	W^2	D^*	D^-	X^2	U	W^2	D^*	D^-	X^2	U	W^2	D^*	D^-	X^2	U	W^2
.7	.1	.1478	.1478	136	.82	33.04	.2038	.2038	152	.90	55.72	.1124	.0101	51	.40	5.24	.1525	.1525	188	.77	43.59
	.05	.1449	.1449	107	.96	35.23	.2576	.2576	212	.99	84.75	.1168	.0000	35	.00	9.35	.1764	.1764	154	.94	59.62
	.02	.1616	.1616	149	.99	55.25	.2783	.2783	269	.99	107.67	.0625	.0018	33	.28	2.76	.1852	.1852	206	.99	90.92
	.01	.1359	.1359	114	.99	40.59	.2644	.2644	280	.99	112.20	.0942	.0000	41	.00	7.80	.2091	.2091	278	.99	114.38
	.005	.1633	.1633	185	.99	58.28	.3003	.3003	300	.99	109.77	.0478	.0305	20	.29	1.12	.2343	.2343	362	.99	139.18
	.0025	.1429	.1429	170	.95	46.12	.2620	.2620	284	.99	93.35	.0671	.0418	27	.34	3.13	.2064	.2064	328	.99	120.09
	1000⁺	.0205	.0205	17	.56	.72	.0384	.0095	21	.17	1.21	.0381	.0381	30	.91	1.25	.0315	.0315	20	.68	.80
.9	.1	.2912	.0278	519	.27	153.54	.2642	.0683	373	.38	82.37	.4212	.0131	255	.22	89.87	.2800	.0311	443	.16	142.32
	.05	.2076	.0157	255	.05	84.28	.1460	.0759	146	.41	24.90	.4028	.0000	262	.22	108.33	.1795	.0331	209	.17	64.66
	.02	.1330	.0019	112	.01	37.67	.0686	.0686	46	.59	5.29	.3108	.0014	217	.07	90.82	.1039	.0189	79	.14	19.94
	.01	.1002	.0000	77	.01	24.49	.0620	.0620	29	.80	3.60	.2494	.0001	159	.44	71.98	.0607	.0083	34	.15	5.46
	.005	.0718	.0000	60	.01	16.00	.0697	.0697	30	.99	5.46	.2152	.0000	142	.03	62.97	.0200	.0200	23	.57	.78
	.0025	.0673	.0059	51	.06	10.67	.1032	.1032	40	.99	9.95	.1893	.0008	138	.01	22.82	.0717	.0717	54	.65	7.08
	1000⁺	.0253	.0253	15	.54	.73	.1414	.0086	69	.03	25.52	.1335	.1335	73	.99	22.97	.0164	.0164	15	.53	.34

⁺ Results for fixed N = 1000 observations.

below. This implies that completion N has either a long service time or a
short interarrival time, manifesting a larger than average X_{N+1} .

An appropriate partition further clarifies this issue. Let $A(a, b) = [a, b]$
for $a \leq b$.   Consider the starting rules

<u>Rule 1a</u> $N = \min(n: \hat{\rho}_n \in A(\rho - \delta, \rho)$ and $\hat{\rho}_i \notin A(\rho, \rho + \delta)$, $i = 1,\ldots,n-1)$

<u>Rule 1b</u> $N = \min(n: \hat{\rho}_n \in A(\rho, \rho + \delta)$ and $\hat{\rho}_i \notin A(\rho - \delta, \rho)$, $i = 1,\ldots,n-1)$.

Together these give rule 1. Figures 4 and 5 show deviations for those of the
1000 replications that satisfy rule 1a and Figures 6 and 7 display deviations
for the remaining replications that satisfy rule 1b. The clarification is
relatively immediate. As δ decreases rule 1a results in negative deviations
and rule 1b results in positive deviations. A little thought provides a
plausible explanation. In order for rule 1a to apply on replication j the
service time for completion N_j tends to be larger than average. Consequently,
the associated system time is larger than average. The net effect is that the
distribution of system times based on rule 1a is <u>stochastically</u> <u>greater</u> than
the steady-state distribution of system time. A one sided test of the Kolmo-
gorov-Smirnov statistics with $\alpha = 0.05$ for rule 1a easily confirms this obser-
vation for all δ and $\rho = .7$ and $.9$. Moreover, the U's provide additional
support. By contrast rule 1b produces system times whose distribution is
stochastically less than the steady-state distribution.

To summarize briefly, rule 1 cannot guarantee a steady-state distribution
as δ decreases. However, rule 1a produces a stochastically greater than
distribution as δ decreases. This behavior implies larger than expected
system times so that rule 1a leads to an overcongested system at the beginning

of data collection. Since the objective is to measure congestion we regard this as a reasonable situation to induce when steady state is unobtainable.

To recommend rule 1b would be somewhat shortsighted. Clearly if $\hat{\rho}_i \in A(\rho, \rho + \delta)$ for i < n one cannot realize the conditions of the rule. To overcome this inadequacy consider

<u>Rule 2</u> $N = \min(n: |\hat{\rho}_n - \rho| \leq \delta$ and $\hat{\rho}_n > \hat{\rho}_{n-1})$.

This rule allows for starting data collection only when $\hat{\rho}_n$ demonstrates an increase over $\hat{\rho}_{n-1}$. But one expects this to happen when completion n+1 has a longer service time or shorter interarrival time than average thereby contributing to an above average system time.

For each value of ρ and δ 1000 replications were run using rule 2 and X_{N+1} was recorded. Table 1 shows the goodness-of-fit statistics. Also, for $\alpha = 0.05$ and $\rho = .7$ a test of the Kolmogorov-Smirnov statistic D^- supports the assertion that rule 2 has induced a "stochastically greater than" distribution for X_{N+1} . For $\rho = .9$ D^- supports this type of behavior only for $\delta = .0025$. Figures 8 and 9 show the deviations.

On the basis of the accumulated empirical evidence to date, one inclines to recommend the use of rule 2 with $\delta = .0025$. Although we do not quarrel with this recommendation, this advice should be regarded as a temporary measure on at least three grounds. Firstly, we have no experience with $\rho > .9$. Secondly, we have no experience with multiserver systems. Thirdly, the sample quantiles of starting time for rule 2 and $\delta = .9$ in Table 3 are cause for concern. Notice that, although 90 percent of the starting times are less than 3099, one percent exceeds 48334. In our opinion the risk of excessive cost is far too great to regard rule 2 as an end in itself. Therefore, research on starting

rules that produce shorter starting times is in order. Section 4 outlines
prospective research on this extension. The motivation for the approach
relies on an empirical observation that as δ decreases the correlation
between N and X_{N+1} becomes small. Although this observation needs
careful study, the preliminary evidence encourages us to consider its impli-
cations for starting rules.

Table 3
Sample Quantiles of Starting Time for Rule 2
$$\rho = .9$$
$$pr(N \leq n) = p$$

						n							
δ \\ p	.01	.02	.05	.10	.25	.50	.75	.90	.95	.98	.99	maximum	mean
.1	2	2	2	4	6	13	31	82	141	242	457	536	33
.05	2	2	4	5	10	22	55	173	335	644	759	1861	74
.02	3	4	6	9	19	43	131	535	1193	2350	4274	10230	248
.01	4	5	9	14	29	69	223	1112	2268	6981	10592	56953	642
.005	4	7	12	18	45	111	400	1917	6318	14098	29718	193990	1586
.0025	4	9	19	34	73	191	717	3099	9670	27512	48334	525550	3471

4. An Iterative Rule

The observation that X_{N+1} and N may be uncorrelated for small δ suggests
an approach to effecting the same result that rule 2 induces at a considerably
smaller cost. Suppose X_{N+1} and N are independent and let

$$q_j = pr(N \leq j) \quad ,$$

based on rule 2. Also define the <u>local</u> sample activity level

$$\hat{\rho}_{I,N} = \frac{\sum\limits_{j=mI+1}^{N} S_j}{\sum\limits_{j=mI+1}^{N} T_j} \cdot \frac{N - mI - 1}{N - mI}$$

where m is a positive integer and I is the integer part of (N-1)/m .
This sample activity level applies for the M/M/1 and may require an adjust-
ment in its correction factor for distributions other than the exponential.

Consider the rule

<u>Rule 3</u> $N^* = \min(n: |\hat{\rho}_{I,n} - \rho| \leq \delta \text{ and } \hat{\rho}_{I,n} > \hat{\rho}_{I, n-1}$

and $mI \neq n-1)$.

In words this rule requires us to use a sample activity level based on at
most m past completions. The quantity I denotes the number of times we
need to reset $\hat{\rho}_{I,N}$; i.e., the number of iterations. A little thought shows
that

$$pr(I = i) = (1 - q_m)^i q_m \qquad\qquad i = 0,1,\ldots \quad .$$

Then I has a geometric distribution with mean $(1 - q_m)/q_m$ and variance
$(1 - q_m)/q_m^2$. Also, the mean number of completions $E(N^*)$ required to
meet rule 3 satisfies

$$m(1 - q_m)/ q_m < E(N^*) \leq m/ q_m \quad .$$

Now a user may choose m to suit one's convenience. However, from the
optimality viewpoint one prefers the m that minimizes m/q_m . If several

m's lead to identical minima then one prefers the largest among them since this minimizes $\text{var}(mI) = (1 - q_m)(m/q_m)^2$. Naturally, the efficacy of rule 3 remains to be verified by careful experimentation. If this verification is realized we will have a considerably more desirable starting time rule than rule 2.

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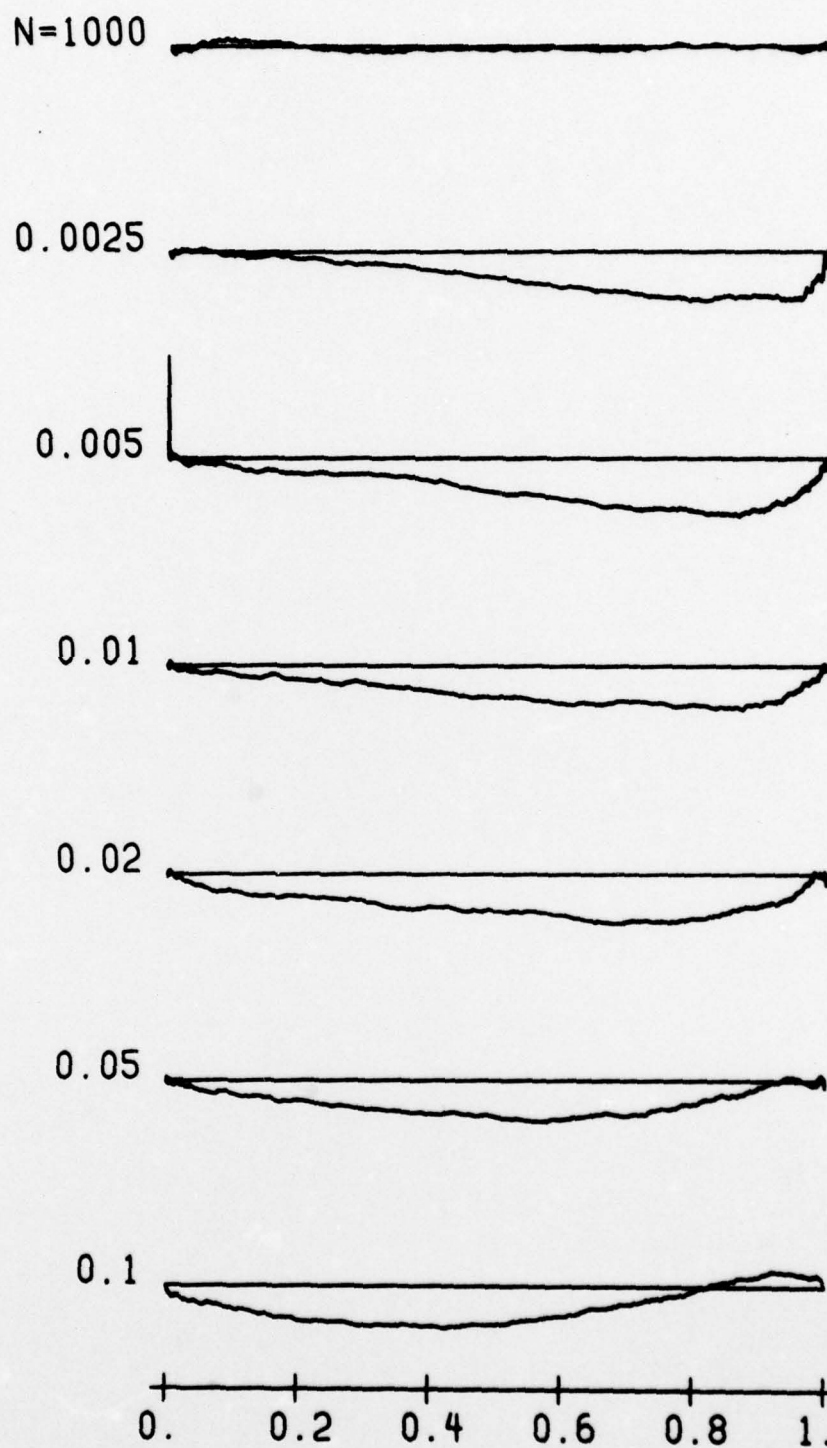
APPENDIX

Fig. 1 $\overline{\Delta}(y)$ vs. y for Rule 1 with $\rho = .7$

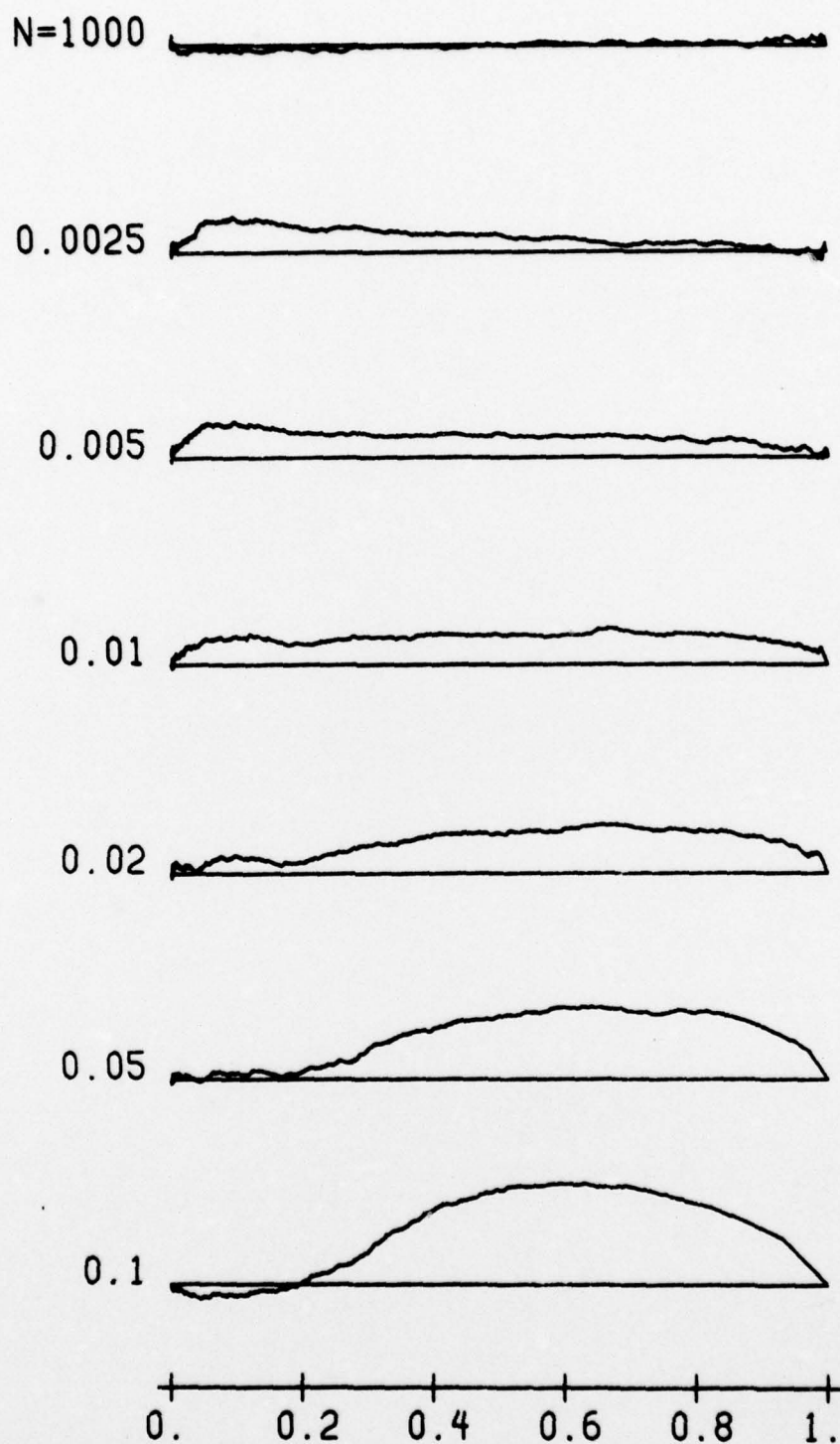$$\overline{\Delta}(y) = \Delta(y)/\sqrt{y(1-y)}$$

Fig. 2 $\overline{\Delta}(y)$ vs. y for Rule 1 with $\rho = .9$
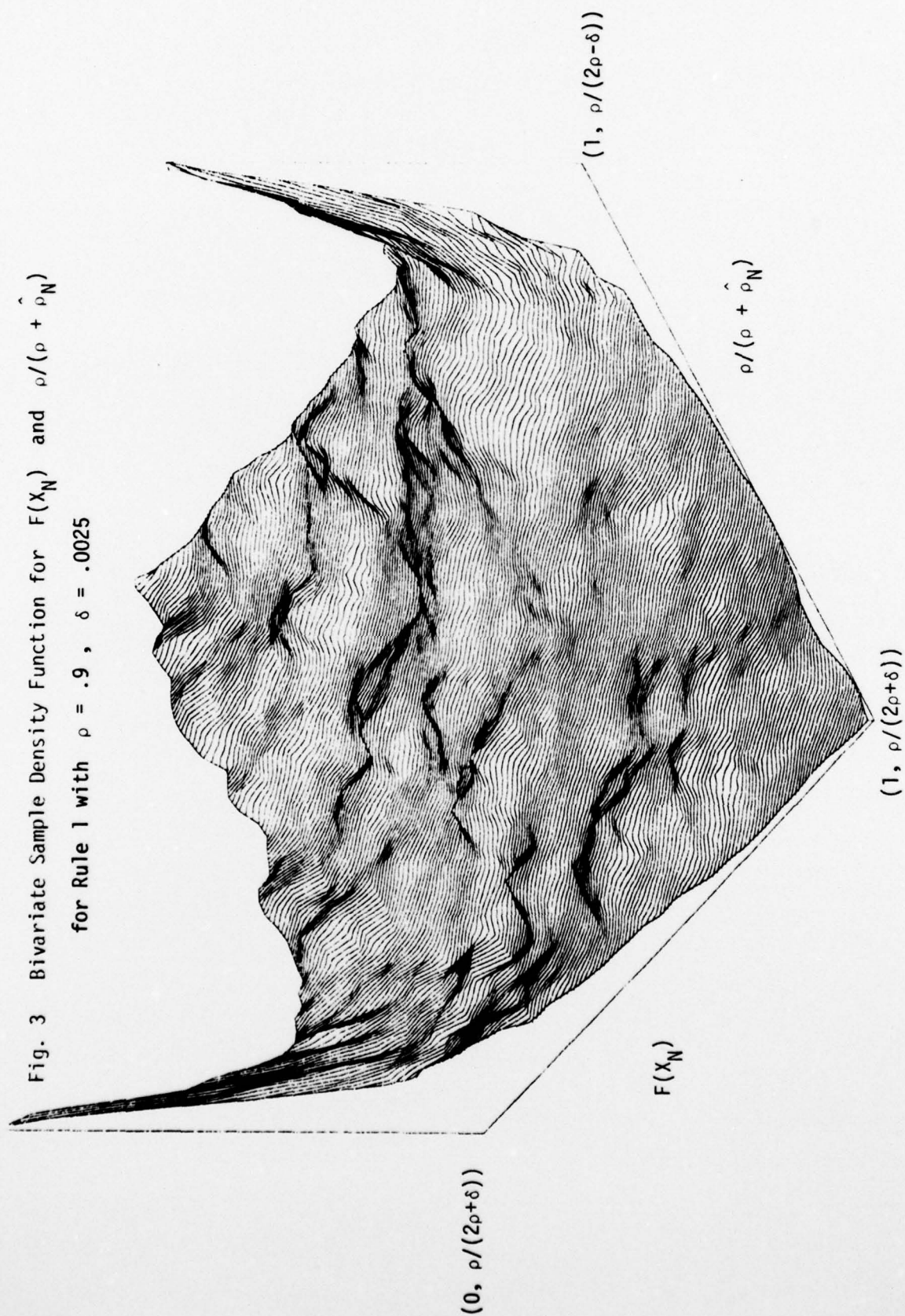
$$\overline{\Delta}(y) = \Delta(y)/\sqrt{y(1-y)}$$

Fig. 3 Bivariate Sample Density Function for $F(X_N)$ and $\rho/(\rho + \hat{\rho}_N)$
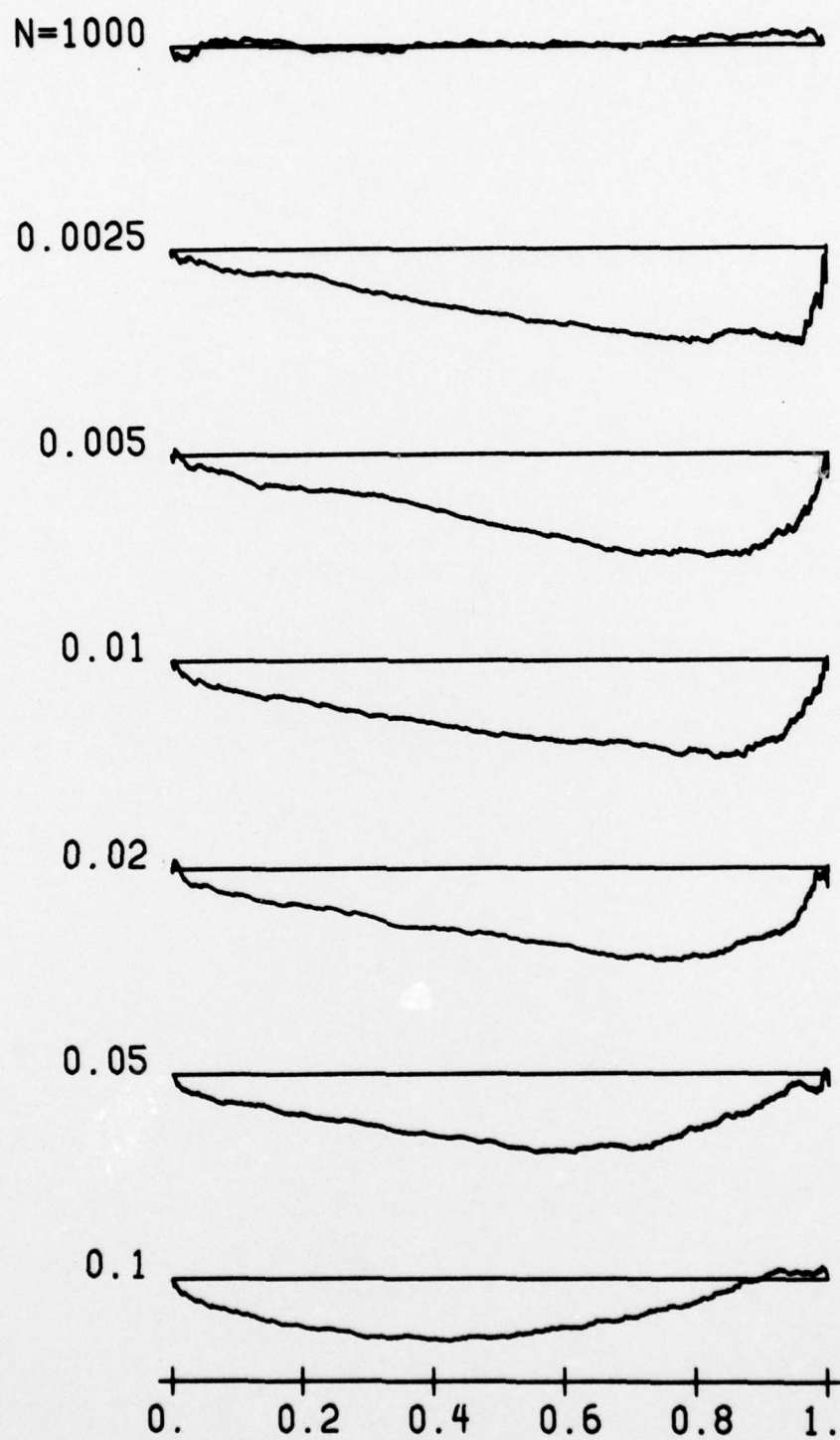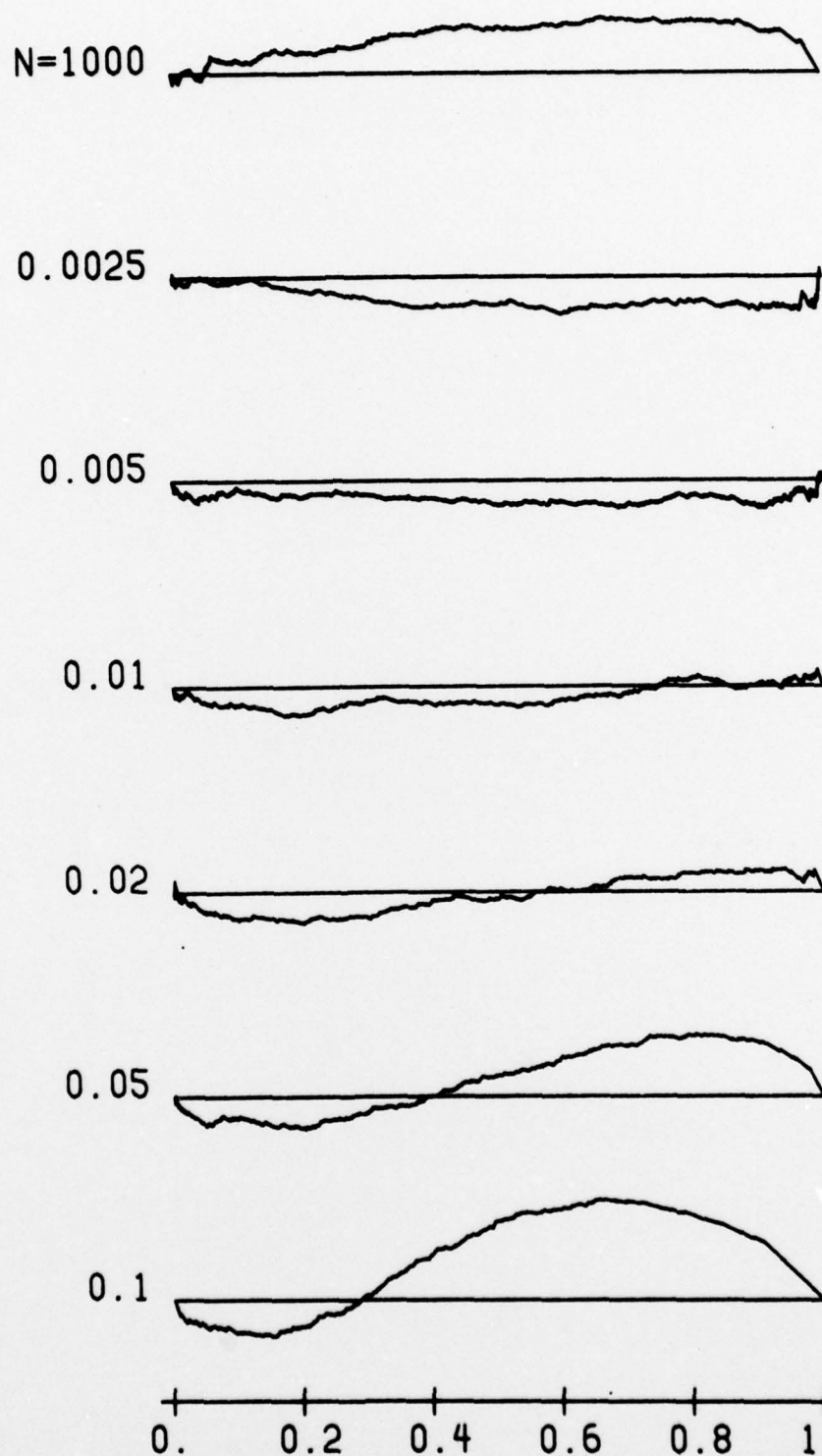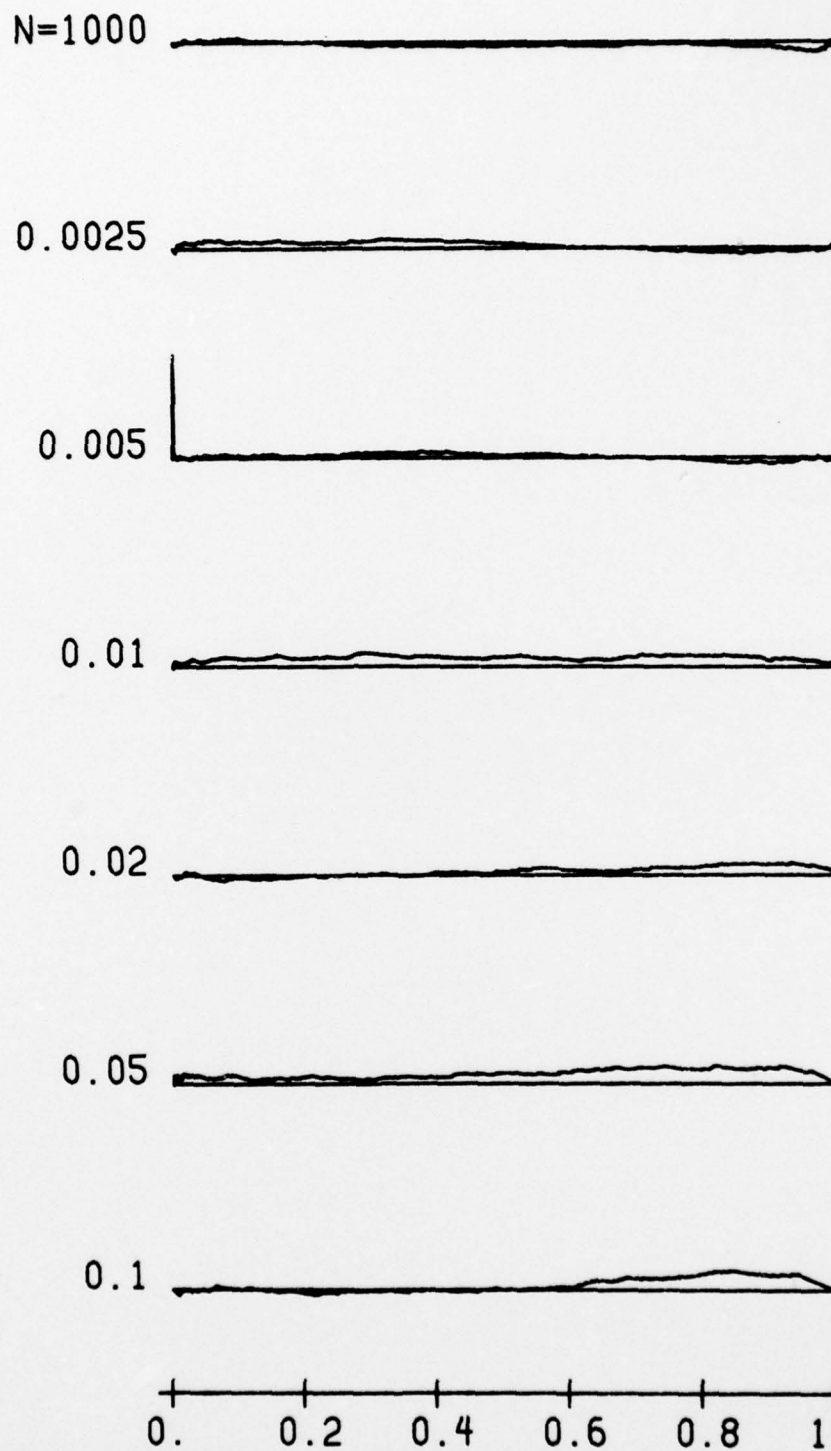
for Rule 1 with $\rho = .9$, $\delta = .0025$

$(1, \rho/(2\rho-\delta))$

$\rho/(\rho + \hat{\rho}_N)$

$(1, \rho/(2\rho+\delta))$

$F(X_N)$

$(0, \rho/(2\rho+\delta))$

Fig. 4 $\overline{\Delta}(y)$ vs. y for Rule 1a with ρ = .7

$\overline{\Delta}(y) = \Delta(y)/\sqrt{y(1-y)}$

Fig. 5 $\overline{\Delta}(y)$ vs. y for Rule 1a with ρ = .9
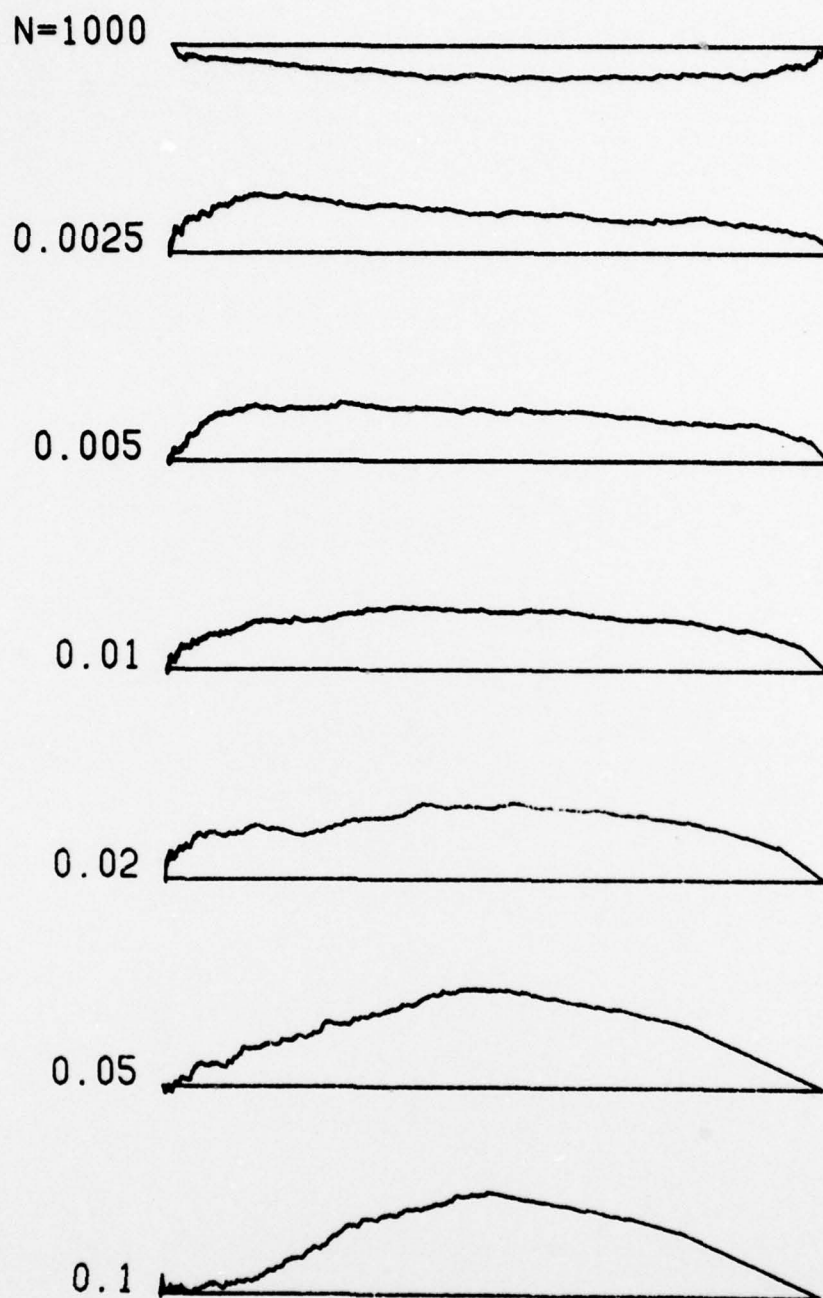
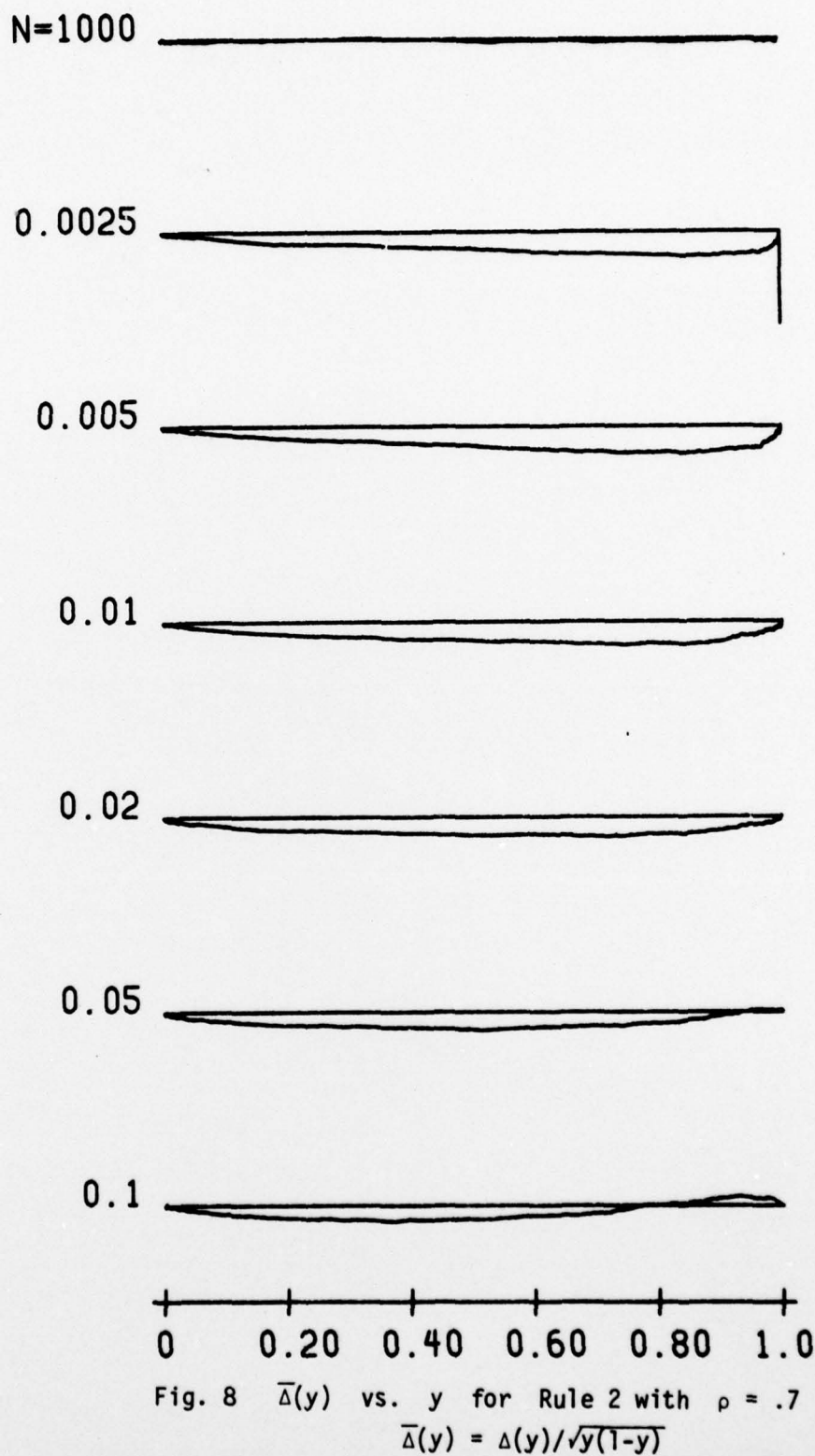$$\overline{\Delta}(y) = \Delta(y)/\sqrt{y(1-y)}$$

Fig. 6 $\overline{\Delta}(y)$ vs. y for Rule 1b with $\rho = .7$

$$\overline{\Delta}(y) = \Delta(y)/\sqrt{y(1-y)}$$

Fig. 7 $\overline{\Delta}(y)$ vs. y for Rule 1b with $\rho = .9$

$$\overline{\Delta}(y) = \Delta(y)/\sqrt{y(1-y)}$$

Fig. 8 $\overline{\Delta}(y)$ vs. y for Rule 2 with $\rho = .7$
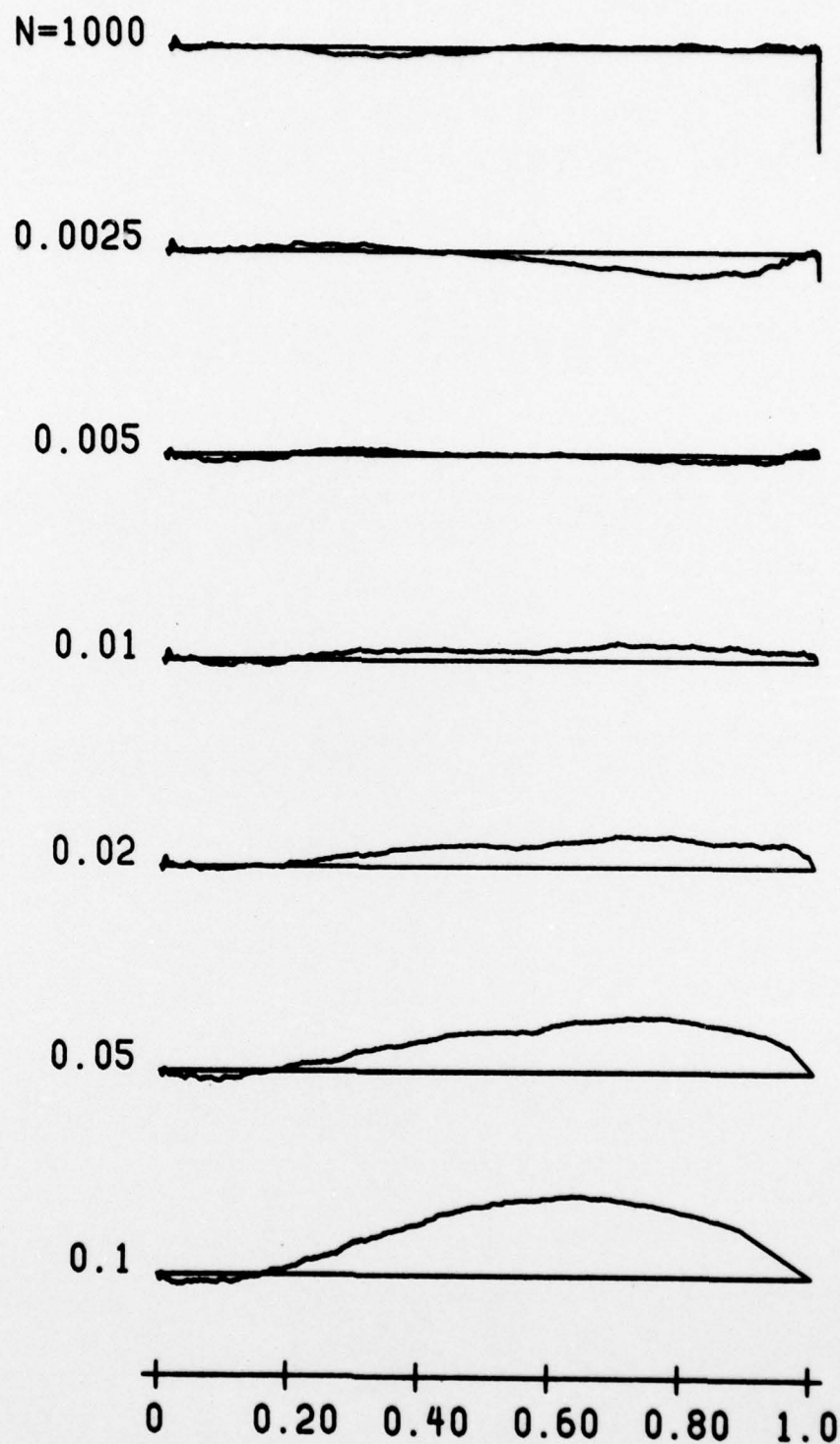
$\overline{\Delta}(y) = \Delta(y)/\sqrt{y(1-y)}$

Fig. 9 $\overline{\Delta}(y)$ vs. y for Rule 2 with $\rho = .9$

$$\overline{\Delta}(y) = \Delta(y)/\sqrt{y(1-y)}$$

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This paper presents results of experimentation aimed at identifying suitable starting rules for discrete event simulations. A starting rule is a decision rule that tells a simulation analyst when to begin collecting data that are relatively free of the initial conditions of a simulation. The starting rules described here rely for decision making on a comparison between a priori information on interarrival and service times and corres-

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20. ponding sample quantities computed during the course of a simulation. Testing of the first proposed rule on a single-server queueing simulation with exponential interarrival and service times revealed a serious inadequacy. However, an examination of just how this inadequacy arose led to a second proposal for a starting rule. When tested in a parallel simulation the second rule produced considerably more favorable results. In addition, a perusal of the distribution of starting time for 1000 replications suggests a direction for future research aimed at reducing this starting time.